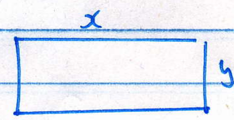
11/08/12

① of all rectangles with a fixed area A, which one has the minimum perimeter?

Ans



Area $A = xy$ Constant A is fixed

P = perimeter $= 2x + 2y$ Goal minimize P

$A = xy \Rightarrow y = \frac{A}{x}$, hence $P = 2x + 2\left(\frac{A}{x}\right)$,

$P(x) = 2x + 2\left(\frac{A}{x}\right)$

As $xy > 0$, we shall have $x > 0$; want to minimize $P(x)$

$$P'(x) = 2 - \frac{2A}{x^2}$$

want $P'(x) = 0 \Rightarrow 2 - \frac{2A}{x^2} = 0 \Rightarrow \cancel{2} = \frac{\cancel{2}A}{x^2} \Rightarrow x^2 = A \Rightarrow x = \sqrt{A}$

By the SDT, since $P''(x) = \frac{4A}{x^3} > 0$, hence $P(x)$ has a local minimum at $x = \sqrt{A}$

when $x = \sqrt{A}$, $y = \frac{A}{x} = \frac{A}{\sqrt{A}} = \sqrt{A}$

$\therefore$ dimensions of the rectangle given by $x = \sqrt{A}$, $y = \sqrt{A}$ to minimize perimeter //

② What two non negative real numbers a and b, whose sum is 23 maximize a^2+b^2? minimize a^2+b^2?

Ans Constant $a + b = 23$, $a \geq 0$, $b \geq 0$ | Goal: maximize a^2+b^2, minimize a^2+b^2

$b = 23 - a$

$Q = a^2 + b^2$, $Q = a^2 + (23-a)^2$

$= a^2 + 529 + a^2 - 46a$, $Q(a) = 2a^2 - 46a + 529$, $0 \leq a \leq 23$

$Q'(a) = 4a - 46$, $Q'(a) = 0 \Rightarrow 4a = 46 \Rightarrow a = \frac{\cancel{46}^{23}}{\cancel{4}_{2}}$; $a = \frac{23}{2}$

$Q''(a) = 4$ | $Q(0) = 529$, $Q\left(\frac{23}{2}\right) = 5 \cdot \frac{29}{2}$, $Q(23) = 529$

Hence absolute maximum occurs when $a = 0$ and $b = 23$

or $a = 23$ and $b = 0$

Absolute minimum occurs at $a = \frac{23}{2}$, $b = \frac{23}{2}$ //